THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 **BASIC APPLIED MATHEMATICS**

(For Both School and Private Candidates)

Time: 3 Hours ***Monday, 8th February 2010 a.m.***

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 marks)

Answer **all** questions in this section.

1. The coordinates for points A, B and C are (2, 9), (4, 3) and (2, -5) respectively. If the line through C with gradient $\frac{1}{2}$ meets the line AB produced at D, find:

   (a) the coordinates of D.

   (b) the equation of the line through D perpendicular to the line $5y - 4x = 17$.

   (6 marks)

2. A function is defined by $f(x) = \frac{1}{1-x}$, $x \neq 1$.

   (a) Why is 1 excluded from the domain of f.

   (b) Sketch the curve $y = f(x)$.

   (c) Find $f^{-1}(x)$ in terms of x and give the domain and range of f^{-1}.

   (6 marks)

3. (a) Which term of the sequence 14, 21, 28,.....is 168?

   (b) In a certain geometric progression, the third term exceeds the first by 9 while the second term exceeds the fourth by 18. Find the numbers.

   (6 marks)

4. (a) If y varies jointly and directly as cube root of x and the square root of z, express this statement as an equation given that y = 2 when $x = \frac{1}{8}$ and $z = \frac{1}{4}$.

   (b) A bank uses the formula $A = P\left(1 + \frac{r}{100}\right)^n$ to calculate the amount of money in an account. Calculate A when $P = 800$, $r = 6$ and $n = 5$ correct to 2 decimal places.

   (6 marks)

5. (a) Given that $\tan 75^\circ = 2 + \sqrt{3}$, find in the form $m + n\sqrt{3}$, where m and n are integers, the value of (i) $\tan 15^\circ$ (ii) $\tan 105^\circ$.

   (b) Find, in radians to two decimal places, the values of x in the interval $0 \leq x \leq 2\pi$, for which $3\sin^2 x + \sin x - 2 = 0$.

   (6 marks)

6. (a) Use the laws of logarithms to express $3\ln 4 - \ln 24 + \frac{1}{2}\ln 2.25$ as a single logarithm in its simplest form, showing all your working.

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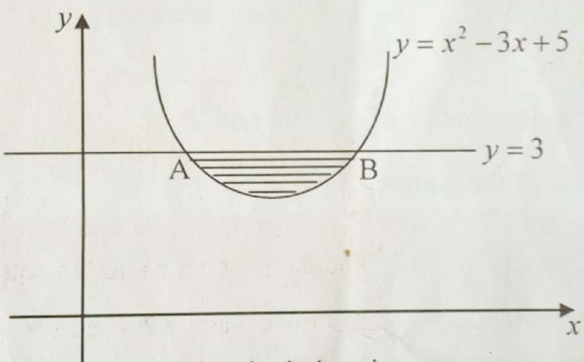
(b) Given that $p = e^x$ and $q = e^y$, express without involving either logarithms or powers of e:
(i) e^{x+y} (ii) e^{2x-y}. **(6 marks)**

7. (a) Find $f'(x)$ from first principles, given that $f(x) = \sqrt{2x+1}$.

(b) If $T = 2\pi\sqrt{\frac{L}{g}}$ where π and g are constants, find $\frac{dL}{dT}$.

(c) A curve is represented parametrically by the equations $x = \frac{t}{1+t}$, $y = \frac{t^3}{1+t}$.
Find $\frac{dy}{dx}$ in terms of t. **(6 marks)**

8. The graph shows sketches of the line $y = 3$ and the curve $y = x^2 - 3x + 5$ they intersect at the points A and B. The shaded region is bounded by the arc AB and the chord AB.



(a) Find the area of the shaded region.

(b) Show that the equation of the tangent to the curve at A is $y + x - 4 = 0$ and find the equation of the tangent to the curve at B. **(6 marks)**

9. Three points P, Q and R have position vectors, $\underline{p}$, $\underline{q}$ and $\underline{r}$ respectively, where $\underline{p} = 7i + 10j$, $\underline{q} = 3i + 12j$, $\underline{r} = -i + 4j$.

(a) Write down the vectors $\overline{PQ}$ and $\overline{RQ}$, and show that they are perpendicular.

(b) If S is the midpoint of PR, show that $|\overline{QS}| = |\overline{RS}|$. **(6 marks)**

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10. The marks obtained by 80 students in an examination are shown below.

Mark	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	90-1
Frequency	3	5	5	9	11	15	14	8	6	4

Plot a cumulative frequency curve and hence estimate:
(a) the median,
(b) the lower and the upper quartile. **(6 marks)**

SECTION B (40 marks)

Answer **four (4)** questions from this section. Extra questions will **not** be marked.

11. (a) A coin is tossed twice. List down the possible outcomes (sample space) and hence find the probability of obtaining at least one head.

(b) Using the sample space (S) in part (a) above, demonstrate that the probability of a sample space is always one (1).

(c) Find the value of n that will satisfy the following equation
$3 \times {}^{n+1}C_3 = 7 \times {}^{n}C_2$. **(10 marks)**

12. (a) Find $(A+B)^2$ given that $A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$

(b) Find the inverse of the matrix

$$C = \begin{pmatrix} -1 & 1 & -2 \\ -2 & 2 & 1 \\ 1 & -2 & -3 \end{pmatrix}.$$

(c) A restaurant sells three meals A, B and C. In two days the sales were as shown in matrix S.

$$S = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cc} \text{Mon} & \text{Tue} \end{array} \\ \begin{pmatrix} 10 & 5 \\ 15 & 20 \\ 10 & 10 \end{pmatrix} \end{array}$$

The price in (\$) paid for each meal is given by matrix P

$$P = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 2 & 4 & 7 \end{pmatrix} \end{array}$$

Work out the matrix product PS and interpret the result. **(10 marks)**

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13. Two mills produce the same three types of plywood. The table given below gives the production, demand and cost data.

Plywood type	**Mill 1 per day**	**Mill 2 per day**	**Six – month demand**
A	100 sheets	20 sheets	2000 sheets
B	40 sheets	80 sheets	3200 sheets
C	60 sheets	60 sheets	3600 sheets
Daily costs	Tshs 3,000,000	Tshs 2,000,000	

Find the number of days that each mill should operate during the 6 months in order to supply the required sheets in the most economical way. **(10 marks)**

14. (a) The gradient of a curve at the point (x, y) is $3x^2 - 4x + 1$. If the curve passes through the point $(2, 3)$:

(i) Show that the curve also passes through the point A(-1, -3),

(ii) Find the equation of the normal at A(-1, -3),

(iii) Find the maximum and minimum values of y.

(b) Use the definition $y + \Delta y = f(x + \Delta x)$, find the cube root of 1010 for $\Delta x = 10$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

(c) Find:

(i) $\int \cos^3 x \, dx$,

(ii) $\int_0^1 \left(\frac{1}{x+2} + \frac{4}{2x+5}\right) dx$. **(10 marks)**

15. (a) Sketch the curve $y = x + x^2$ for $-2 \le x \le 4$ and shade the area bounded by the curve, the x – axis and the lines $x = 2$ and $x = 3$.

(b) Find the volume of the solid of revolution obtained by rotating the shaded area in part (a) above.

(c) Find the coordinates of the points on the curve $y = x^3 - 3x$ at which the tangent is parallel to the x – axis. **(10 marks)**

$x^2 - 3 = 0$

$3x^2 - 3 = 0$

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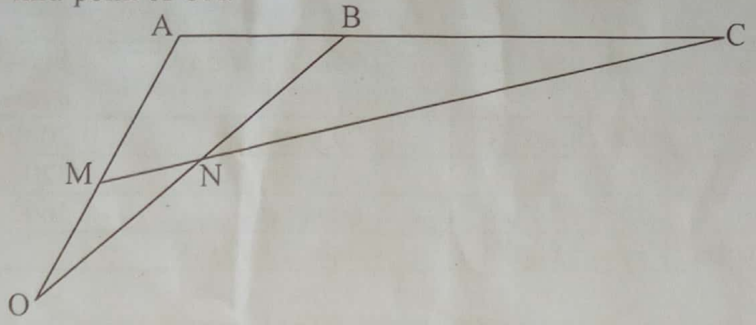
16. (a) In the diagram below, O is the origin, ABC is a straight line and M is the mid-point of OA.



If $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$ and $\overline{AC} = 3\overline{AB}$, find in terms of $\bar{a}$ and or $\bar{b}$, in their simplest form:

(i) $\overline{MA}$ (ii) $\overline{AC}$ (iii) the position vector of C.

(b) A, B and C are the points with position vectors $2i - j + 5k$, $i - 2j + k$ and $3i + j - 2k$ respectively. If D and E are the respective midpoints of BC and AC, show that DE is parallel to AB.

(c) If $4\begin{pmatrix}1\\3\end{pmatrix} + 2\begin{pmatrix}1\\m\end{pmatrix} = 3\begin{pmatrix}n\\-6\end{pmatrix}$ find the values of m and n. **(10 marks)**

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